13.3 day 2

Vertex Formula

Applications

$y = 1x^2 - 4x + 1$

$y - 1 + 4 = x^2 - 4x + 4$

$y + 3 = (x-2)^2$

$y = {}^{+}1(x-2)^2 - 3$

Vertex Form

$y = a(x-h)^2 + k$

$V(h, k)$

$V(2, -3)$ min

$y = {}^{+}1(x-2)^2 - 3$

V $(2, -3)$ min

axis of symm.

$x = 2$

X-intercepts

$0 = (x-2)^2 - 3$

$\pm\sqrt{3} = \sqrt{(x-2)^2}$

$\pm\sqrt{3} = x - 2$

$2 \pm \sqrt{3} = x$

x < $2+\sqrt{3} \simeq 3.7$; $2-\sqrt{3} \simeq 0.3$

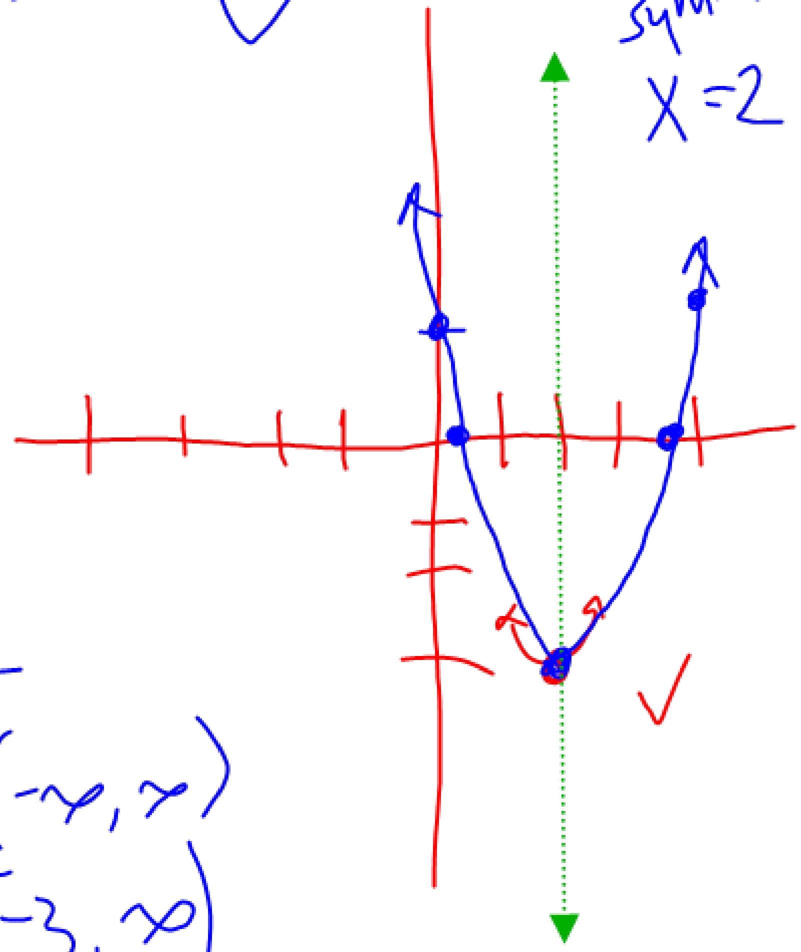
y-int

$x = 0$

$y = 1$

Domain $(-\infty, \infty)$

Range $[-3, \infty)$



$y = 1x^2 - 4x + 1$

find x-intercept

Quadratic Form

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

discriminant

$$x_v = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$$

x	y
2	-3

$y_v = 2^2 - 4(2) + 1$

$= -3$

$v(2, -3)$

Vertex Formula

$$x = \frac{-b}{2a}$$

$$y = {}^{+}3x^2 - 4x + 1$$

find the vertex using the vertex formula, decide min/max

$$x_v = -\frac{b}{2a} \quad \frac{-(-4)}{2(3)} = \frac{2}{3} \qquad V\left(\frac{2}{3}, -\frac{1}{3}\right)_{min}$$

$$y_v = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 1 = -\frac{1}{3}$$

$y = -2x^2 - 4x + 5$

$-2 \quad +4$

$v(-1, 7)$

1) vertex using vertex formula $x = \frac{-b}{2a} = \frac{-(-4)}{2(-2)} = -1$

2) min/max max

$y = 7$

3) Range $(-\infty, 7]$

55. A person standing close to the edge on the top of a 160-foot building throws a baseball vertically upward. The quadratic function

$$s(t) = -16t^2 + 64t + 160$$

models the ball's height above the ground, $s(t)$, in feet, t seconds after it was thrown.

a. After how many seconds does the ball reach its maximum height? What is the maximum height?

b. How many seconds does it take until the ball finally hits the ground? Round to the nearest tenth of a second.

a) $t_v = \frac{-b}{2a} = \frac{-64}{2(-16)} = 2$ sec

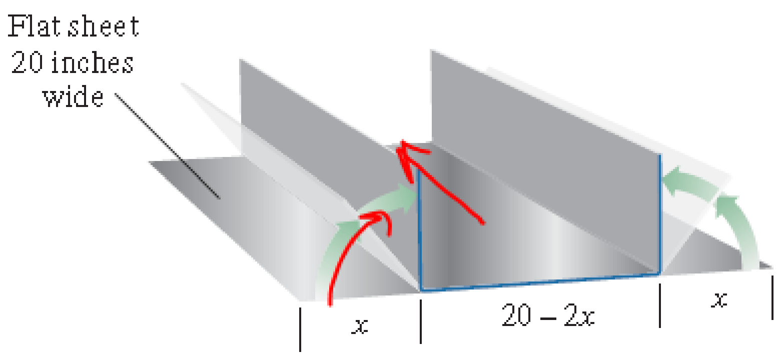
$S(2) = -16(2)^2 + 64(2) + 160 = 224$ ft

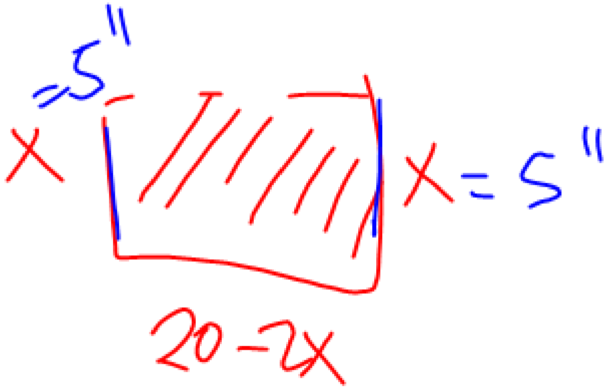
b) $\frac{0}{-16} = \frac{-16t^2}{-16} + \frac{64t}{-16} + \frac{160}{-16}$

$0 = t^2 - 4t - 10 \begin{cases} 10 \cdot 1 \\ 2 \cdot 5 \end{cases}$

$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)} \approx$ ~~10 sec~~ 5.7 sec

67. A rain gutter is made from sheets of aluminum that are 20 inches wide by turning up the edges to form right angles. Determine the depth of the gutter that will maximize its cross-sectional area and allow the greatest amount of water to flow. What is the maximum cross-sectional area?

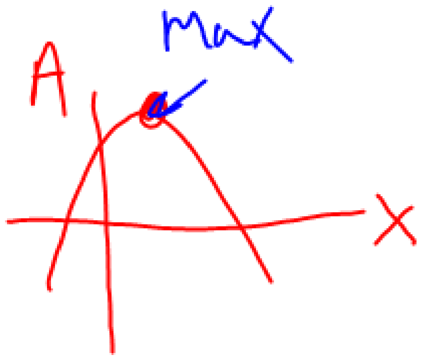




$$A = x(20 - 2x)$$

$$A = 20x - 2x^2$$

$$A = -2x^2 + 20x$$

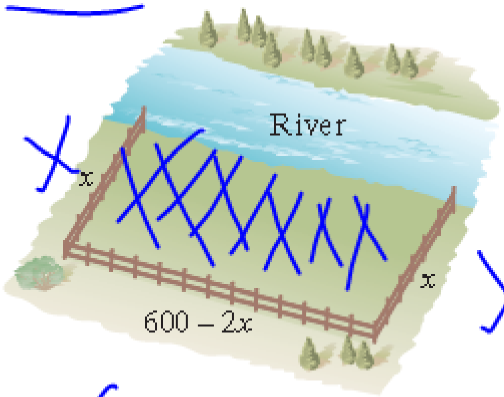


$$x = \frac{-b}{2a}$$

$$x = \frac{-20}{2(-2)} = 5\text{ in}$$

$$A = -2(5)^2 + 20(5) = 50\text{ in}^2$$

63. You have 600 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?



$x = 150'$

$600 - 2x$

$300'$

Dimensions are $150' \times 300'$

Area $= 45{,}000 \text{ ft}^2$

$x_v = -\frac{b}{2a}$

$x_v = -\frac{600}{2(-2)} = 150 \text{ ft}$

$A = x(600 - 2x)$

$A = -2x^2 + 600x$

these functions to solve Exercises 69–70.

69. Hunky Beef, a local sandwich store, has a fixed weekly cost of \$525.00, and variable costs for making a roast beef sandwich are \$0.55.

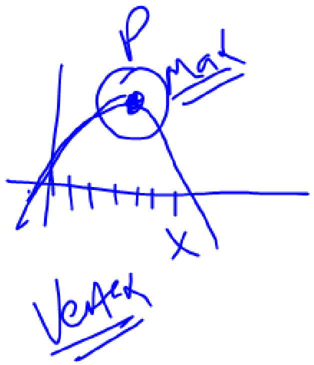
a. Let x represent the number of roast beef sandwiches made and sold each week. Write the weekly cost function, C, for Hunky Beef.

b. The function $R(x) = -0.001x^2 + 3x$ describes the money that Hunky Beef takes in each week from the sale of x roast beef sandwiches. Use this revenue function and the cost function from part (a) to write the store's weekly profit function, P.

c. Use the store's profit function to determine the number of roast beef sandwiches it should make and sell each week to maximize profit. What is the maximum weekly profit?

a) $C(x) = 525 + .55x$

$R(x) = -0.001x^2 + 3x$



$P(x) = R(x) - C(x)$

$P(x) = -0.001x^2 + 3x + -(525 + .55x)$

$P(x) = -0.001x^2 + 2.45x - 525$

$X_v = \dfrac{-2.45}{2(-0.001)} = 1225$ sandwiches

$P(1225) = -0.001(1225)^2 + 2.45(1225) - 525$

$= \$975.62$ per week